Brief Addendum to *ASR* Discussion on Associative Diffusion

Daniel DellaPosta
The Pennsylvania State University
djd78@psu.edu

December 17, 2021

Disclaimer: The referenced comment by DellaPosta and Davoodi (2021, ASR) was peer-reviewed. This brief addendum by the first author was not.

Goldberg's thoughtful reply to the *ASR* comment by Marjan Davoodi and I raises many interesting points to consider. The purpose of this brief addendum is not to respond to all of these, but instead to raise one specific point that seems especially pertinent when considering the whole of the discussion on associative diffusion. First, Goldberg aptly points out that rather than exactly replicating the original associative diffusion model from his 2018 article with Sarah Stein, we in fact ended up creating a slightly different version of the model. Ironically, this unintentional difference between the models likely led to a richer discussion, because it allowed Goldberg to highlight another model parameter that plays an unexpectedly central role in determining the path of associative diffusion processes: the way in which agents update their preferences in order to increase constraint satisfaction (i.e. the fit between their preferences and the cognitive associations they hold about the relationships among those preference dimensions).

In Goldberg and Stein's original model, agents update their preferences by summing their current preference on a given dimension and a randomly drawn number from a normal distribution with a mean of 0 and a variance of 1. In the DellaPosta and Davoodi version, agents instead draw the random number from a uniform distribution bounded by -1 and 1. In both cases, agents only accept the updated preference if doing so would increase their constraint satisfaction; otherwise, they stick with their existing preference. Goldberg and Stein's model of associative diffusion is a theoretical model, not one that draws on empirical data. Accordingly, as Goldberg says in his reply, our assumption is "not categorically incorrect" but simply "an inherently different assumption than the one informing the original AD model." However, Goldberg conjectures that the original approach (using a normal distribution) is preferable because it "implies that only a handful of agents dramatically update their preferences," whereas the revised approach (using a uniform distribution) implies "that the magnitude of agents' updates is equally likely to be large as it is to be moderate." However, subsequent consideration would suggest that this characterization is in fact not true of the two models.

In the uniform-distribution approach, the magnitude of agents' updates are equally likely to be anywhere in the range 0 to 1 (and these updates can be positive or negative). However, the magnitude of these updates is also strictly bounded by 0 and 1. In contrast, with the normal-distribution approach favored by Goldberg, agents' updates are drawn from an unbounded normal distribution with a fairly large standard deviation; the values drawn from this distribution will routinely exceed 1 in magnitude (and can again be positive or negative).

To observe for yourself, you could run simple code in R to compare the resulting distributions. If you create a uniform distribution by running the code "runif(X, min = -1, max = 1)" where X is a large number (say, one million), you will end up with a distribution of values whose average magnitude will be about .50. Compare this to the random-normal distribution you obtain by running the code "rnorm(X, mean = 0, sd = 1)". In addition to the obvious mean of 0 and standard deviation of 1 you would expect in the resulting distribution, the average magnitude of the values in the distribution will be about .80. The reason for the larger average magnitude compared to the random-uniform distribution is simply that the large standard deviation and lack of boundedness in the normal distribution produces many values either lower than -1 or greater than 1.

In short, it is the normal distribution favored by Goldberg, more than the uniform distribution used by Davoodi and myself, that makes agents likelier to make large rather than moderate changes to their preferences. By Goldberg's conjecture, our approach would seem to be the more preferable among the two options.

These two approaches, of course, do not exhaust the possibilities. Like Goldberg, I also come out of this discussion excited for further development of the associative diffusion model so wonderfully described by he and Sarah Stein in their original piece. Goldberg raises many other interesting points in his reply. But on these points, I would prefer to simply let the comment by Davoodi and I speak for itself. I am posting this addendum alongside a direct link to Goldberg's reply in the hopes that interested readers will read the entire exchange, since I think doing so will leave said reader with a rich understanding of this important model and idea.